

# Algorithm for Solar Radiation on Mountain Slopes

LLOYD W. SWIFT, JR.

*Coweeta Hydrologic Laboratory, Southeastern Forest Experiment Station, Forest Service  
U.S. Department of Agriculture, Franklin, North Carolina 28734*

A generalized algorithm provides the daily total of potential solar radiation on any sloping surface at any latitude. The algorithm can be coded as subroutines of a computer model that requires solar radiation as a variable. The required inputs are Julian dates and the latitude, inclination, and aspect of the slope. In addition to computing potential solar radiation, the routine provides estimates of actual radiation on any slope on the basis of measured solar radiation for a nearby horizontal surface that has the same cloud cover.

Solar radiation received by a watershed is the energy source for evaporation, for heating of the soil, plant, and air mass, for circulation of the air, and for causing and controlling the growth and activity of plants and animals. For this reason, solar radiation may be a prime variable in models of watershed processes or biological activity. Some streamflow models require the daily total of solar radiation as an input, but data on solar energy for mountain slopes are seldom available. One alternative is to use the tables of potential solar radiation by *Frank and Lee* [1966] or those by *Buffo et al.* [1972]. These simplify hand computations, but extensive tables are a cumbersome way of including data on daily radiation for various slopes in a general model.

Furthermore, potential radiation cannot always be used because it is a theoretical value representing the solar radiation which would irradiate a point on the surface of the earth if its atmosphere were not present. *Garnier and Ohmura's* [1968] technique for calculating solar radiation on any slope accounts for the effects of varying optical air masses during the day and uses iterative summation to obtain a daily total. Their method calculates a type of potential radiation, since cloud cover is not considered. In my method, *Okanoue's* [1957] integrated equation as given by *Lee* [1963] is used to calculate the daily total of potential solar radiation on any sloping surface. Measurements of solar radiation are then adjusted by factors calculated from potential radiation to derive estimates of actual radiation on slopes. The mean effect of the cloud cover of each day is thereby included in a slope estimate.

Previous authors have limited the application of *Okanoue's* equation to latitudes between 60°N and 60°S. Two levels of complexity are given here. The basic equations are sufficient for most slopes in the equatorial and temperate regions of the earth but fail for steep slopes or for slopes at higher latitudes. Supplemental functions can be added to make the algorithm completely general for all surfaces at any latitude. The computations are presented as detailed flow charts. In addition to calculating the irradiation of a mountain slope, the algorithm may also be used for detailed calculations such as for individual leaves or for roofs and walls of structures.

## THE ALGORITHM

The first flow chart (Figure 1) shows in detail the computational steps for calculating the daily totals of potential solar radiation and for estimating the actual solar radiation on a slope which is not near the polar region. The symbols used in the flow chart are defined at the end of this article. Times of sunrise and sunset on the slope plus several other accessory

values are calculated. The only inputs necessary for the calculations are the latitude, the slope inclination and aspect, and the day of the year (Julian date). An optional input is a value of the measured solar radiation on a horizontal surface for each date. Measured solar radiation is a daily total in gram calories per square centimeter. The flow charts present equations in which all angle values are in decimal degrees. The calculations represented in Figure 1 are simplifications of the general case in order to allow the use of desk or pocket calculators.

Figure 2 is the flow chart for the alternative routines needed to handle all possible sloping surfaces. They are required for a poleward-facing slope when the sum of the slope inclination plus the absolute value of the latitude of the slope exceeds 66°. These routines avoid anomalous results in solving for the time shift ( $L_2$ ) and selecting the sunrise and sunset times which occur when the calculated location of the equivalent slope is near one of the poles or on the other side of the globe from the actual slope site. The alternative routines make the algorithm completely general for any slope and thus would be preferred for computer routines where programing is not limited by machine size.

Computations are begun at 'start' in Figure 1 by inserting the three characteristics of the mountain slope: inclination ( $I$ ), aspect ( $A$ ), and latitude ( $L_0$ ). The inclination of the actual slope is the positive angle between the plane of the slope and the horizontal plane. The azimuth is the positive angle measured clockwise from north to a projection of the normal to the slope plane. South of the equator, latitudes are negative. For each slope, two auxiliary values, latitude ( $L_1$ ) and time shift ( $L_2$ ), of the equivalent slope are calculated. *Lee* [1963] provides a discussion of the theory of the equivalent slope and the development of the equations used here. *Buffo et al.* [1972] illustrate the geometry of the equations. The denominator for  $L_2$  is zero when  $L_1$  is  $\pm 90^\circ$ , and is negative when the equivalent slope is on the other side of the earth. The alternative calculations for  $D_1$  and  $L_2$ , given in Figure 2, sort out and handle these special cases.

The main body of the computation begins at return point 1, where Julian date ( $J$ ) and measured radiation on a horizontal surface ( $R_h$ , optional) are inserted. Once the calculations are set up for a particular slope, the routine provides data about the radiation on the slope for the specified day of the year and can repeatedly return to accept new inputs of dates and measured solar radiation. If potential solar radiation is the only output desired or if horizontal measurements of solar radiation are not available, the input and calculations with measured radiation ( $R_2$ ) are omitted.

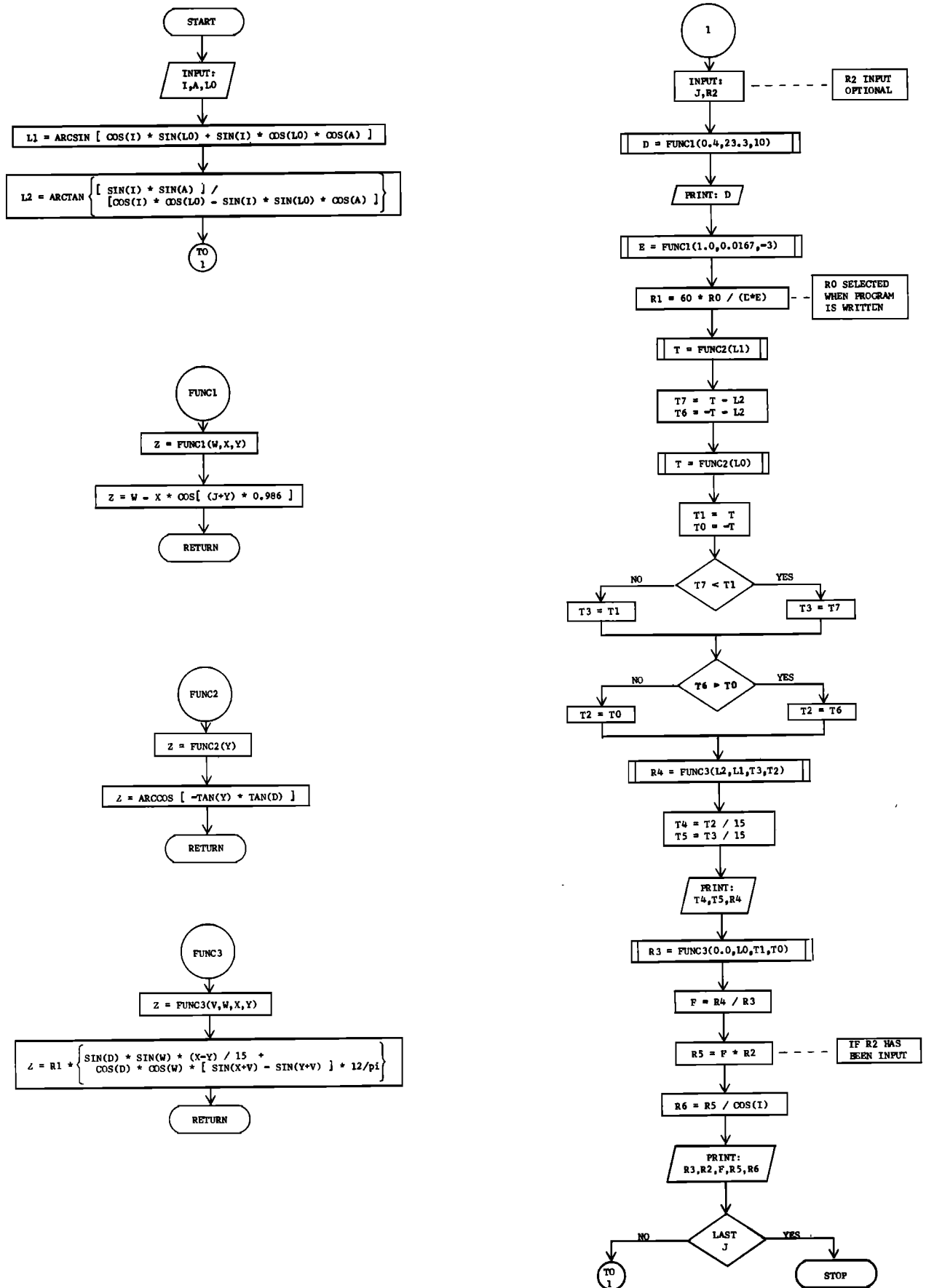


Fig. 1. Flow chart of algorithm for basic solar radiation.

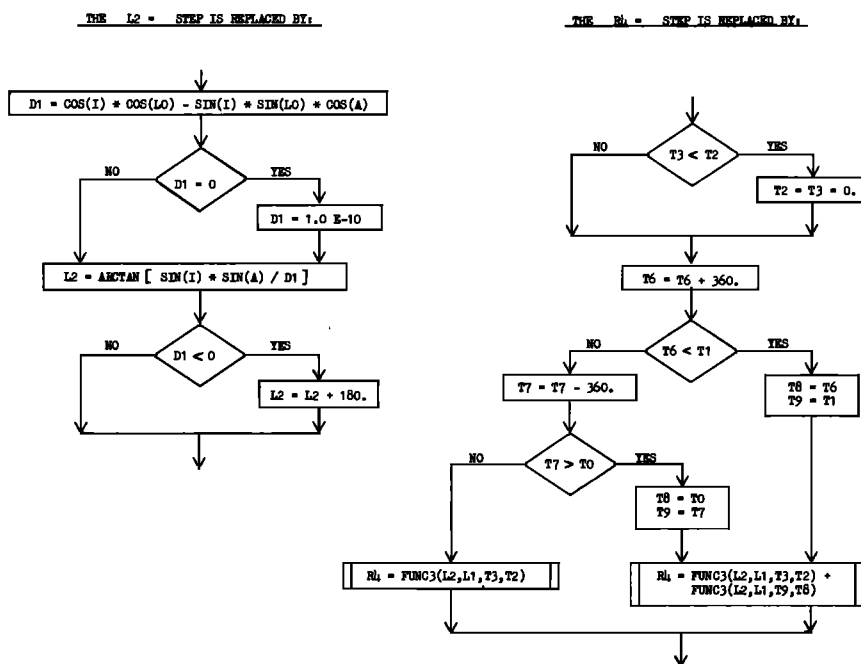


Fig. 2. Flow chart of alternative routines.

Three subroutines, or function operations, are indicated in Figure 1.  $FUNC_1$  provides estimates of the declination and radius vector of the sun as functions of Julian date. Calculating the position of the sun avoids the need for the solar tables required by other algorithms [Fribourg, 1972; Frank and Lee, 1966]. The coefficients for  $FUNC_1$  were derived from the data given in Table 169 by List [1951], except that declinations were advanced to approximate the position at mean solar noon. This simple solar declination equation is not exact, being  $0^\circ 44'$  low in mid-October. With its use, however, estimates of potential solar radiation usually have less than 3% error. Estimates for some steep northward-facing slopes may be low by as much as 16 cal/cm<sup>2</sup>/day in October. Shifting the day number by one, as occurs in leap year, causes the estimate of solar radiation to change by less than 3%. If a complex routine can be used, solar declinations ( $D$ ) accurate to  $\pm 0^\circ 04'$  will be produced by (1), adapted from the *U.S. Naval Observatory* [1945]:

$$D = \arcsin \{0.39785 * \sin [278.9709 + 0.9856 * J + 1.9163 * \sin (356.6153 + 0.9856 * J)]\} \quad (1)$$

For radians the radius vector ( $E$ ) and the declination ( $D$ ) equations should be

$$E = 1.0 - 0.0167 * \cos [(J - 3) * 0.0172] \quad (2)$$

$$D = 0.007 - 0.4067 * \cos [(J + 10) * 0.0172] \quad (3)$$

$$D = \arcsin \{0.39785 * \sin [4.868961 + 0.017203 * J + 0.033446 * \sin (6.224111 + 0.017202 * J)]\} \quad (4)$$

$R_1$  is determined within the main body of the computations to be the solar constant for a 60-min period adjusted for the eccentricity of the orbit of the earth. The expression  $1/E * E$  in the  $R_1$  calculation is slightly more exact than the  $(a/r)^2$  equation given by McCullough and Porter [1971], although both are within 0.3% of the values taken from the work of List [1951]. The user may fix the value of the solar constant (variable  $R_0$ )

when the program is coded. Frank and Lee [1966] used 2.00 cal/cm<sup>2</sup>/min, but Drummond *et al.* [1968] suggests 1.95 cal/cm<sup>2</sup>/min. Varying the solar constant by this amount reduces the potential radiation by 2.5% but does not change the slope factor or the estimate of solar radiation on the slope ( $R_s$ ).

$FUNC_2$ , the second function, begins the calculation of sunrise and sunset times and is used with both the latitudes of the equivalent and the actual slopes:

$$T = \arccos [-\tan(\text{latitude}) * \tan(D)] \quad (5)$$

Since either latitude  $L_0$  or  $L_1$  might be exactly  $90^\circ$  or the argument for the arc cosine might lie outside  $\pm 1.0$ , these values should be tested in  $FUNC_2$  to avoid calculating undefined functions.

Most time variables are hour angles measured from solar noon. By convention, morning values are negative. After the times are selected, the angular velocity of the earth of  $15^\circ/\text{h}$  (0.2618 rad) is used to calculate the decimal hours between solar noon and sunrise ( $T_4$ ) or sunset ( $T_5$ ). This conversion is for the convenience of the user and is not required for other calculations. Further conversion from solar to local standard time requires the time difference between the meridian of the time zone and the longitude of the slope site plus the equation of time, available from Table 169 by List [1951] or from equations by the *U.S. Naval Observatory* [1945].

The decision chain in Figure 1 for selecting  $T_2$  and  $T_3$  is adequate for most slopes. Steep slopes facing away from the equator may not receive any direct radiation on winter days or may be exposed to multiple radiation periods with two sunrises and two sunsets each day. Negative values of potential solar radiation ( $R_s$ ) can result. The supplemental routine of tests and calculations given in Figure 2, when substituted for the  $R_s$  calculation shown in Figure 1, will sort out and correctly solve for all possible slope situations.

Variables  $R_3$  and  $R_4$  give the potential solar radiation for a horizontal surface and for the mountain slope.  $FUNC_3$  is the integrated equation used for both calculations:

TABLE 1. Values Calculated for 3 Sample Days for Three Slopes at 35°N Latitude

Variables	June 23			September 20			December 22		
	30° Westward	30° Southward	65° Northward	30° Westward	30° Southward	65° Northward	30° Westward	30° Southward	65° Northward
<i>Input Values</i>									
<i>I</i> , deg	30	30	65	30	30	65	30	30	65
<i>A</i> , deg	270	180	0	270	180	0	270	180	0
<i>J</i> , day	174	174	174	263	263	263	356	356	356
<i>R</i> <sub>2</sub> , cal/cm <sup>2</sup> /day	634	634	634	341	341	341	221	221	221
<i>Output Values</i>									
<i>D</i> , deg	23.44	23.44	23.44	1.46	1.46	1.46	-23.44	-23.44	-23.44
<i>T</i> <sub>4</sub> , h	-4.61	-6.15	-7.18	-3.71	-6.01	-6.07*	-2.70	-4.82	
<i>T</i> <sub>5</sub> , h	7.18	6.15	7.18	6.07	6.01	-5.44*	4.82	4.82	
<i>R</i> <sub>4</sub> , cal/cm <sup>2</sup> /day	926	839	506	718	886	2	399	764	0
<i>R</i> <sub>5</sub> , cal/cm <sup>2</sup> /day	991	991	991	747	747	747	395	395	395
<i>F</i>	0.935	0.846	0.510	0.961	1.187	0.003	1.008	1.932	0.000
<i>R</i> <sub>6</sub> , cal/cm <sup>2</sup> /day	593	537	324	328	405	1	223	427	0
<i>R</i> <sub>6</sub> , cal/cm <sup>2</sup> /day	684	619	765	378	467	2	257	493	0

\*Double day; second sunrise and sunset are at 5.44 and 6.07.

$$R_4 = R_1 \{ \sin(D) \sin(L_1) (T_3 - T_2) / 15 \\ + \cos(D) \cos(L_1) [\sin(T_3 + L_2) \\ - \sin(T_2 + L_2)] * 12 / \pi \} \quad (6)$$

If calculations are done in radians instead of degrees, the division by 15 in *FUNC*<sub>3</sub> would instead be a multiplication by 3.8197 or  $12/\pi$ . Potential solar radiation has been used by Lee [1963, 1964], Lee and Baumgartner [1966], Jackson [1967], Hendrick *et al.* [1971], and Meiman *et al.* [1971] as an index value to compare the relative energy available to various mountain slopes. Swift and Knoerr [1973] showed that a slope factor, the ratio ( $R_4/R_5$ ) of the potential solar radiation for a slope surface to that for the horizontal surface, could be used to estimate actual solar radiation. When the solar radiation measured on a nearby horizontal surface ( $R_2$ ) is multiplied by the slope factor ( $F$ ), the product (variable  $R_6$ ) is an estimate of solar radiation per unit area of sloping surface. This estimate includes the effects of cloud cover and atmospheric transmissivity that were recorded on the horizontal surface at the climatic station. When measured radiation is not available, the ratio  $F$  can be used as an index of the relative energy available to adjacent slopes, as was proposed by Swift [1960] and Fribourg [1972].

Frequently, a watershed model expresses variables such as streamflow and precipitation on the basis of map area of the surface. For mountain slopes, map area is less than the surface area; thus the radiation intensity is greater per unit of map area than per unit of surface area. Division by the cosine of the slope inclination produces variable  $R_6$ , which is the estimate of the daily total of solar radiation on the basis of map area. Because the intensity of radiation on any slope that faces toward the sun is greater than that on the horizontal surface where solar radiation is measured, the adjustment for map area can result in a valid value of  $R_6$  which is greater than  $R_2$ . The calculation of  $R_6$  for vertical or near-vertical surfaces is meaningless.

As an example of the output from this algorithm, Table 1 lists values calculated for 3 sample days for a 30° slope facing west, a 30° slope facing south, and a 65° slope facing north. All three slopes are at 35°N latitude, and the solar constant used was 1.95.

## CONCLUSION

This algorithm for the solar irradiation of mountain slopes is based on the equations given by Lee [1963] and Frank and Lee [1966]. The method is simplified by eliminating the requirement for tables of solar declination and radius vector. Supplemental routines are also given to make the algorithm general for any sloping surface on the globe. In addition to calculating potential solar irradiation for any slope, the algorithm can use measured solar radiation for a horizontal surface to estimate actual solar radiation on any nearby slope. Programs written in Fortran 4 and in Reverse Polish Notation for a Hewlett-Packard desk calculator may be obtained from the author. (Trade names are given for the convenience of the reader and do not imply endorsement by the Forest Service or by the U.S. Department of Agriculture.)

## NOTATION

- A* azimuth of slope, degrees from north.
- D* declination of sun, positive values are north, negative values are south.
- D*<sub>1</sub> temporary variable, denominator of  $L_2$  equation.
- E* radius vector of sun.
- F* slope factor.
- I* inclination of slope, degrees above horizontal.
- J* Julian date, days from January 1.
- L*<sub>0</sub> latitude of actual slope, positive values are north, negative values are south, degrees.
- L*<sub>1</sub> latitude of equivalent slope.
- L*<sub>2</sub> time offset in hour angle between actual and equivalent slopes.
- R*<sub>0</sub> solar constant, cal/cm<sup>2</sup>/min.
- R*<sub>1</sub> solar constant for 60 min.
- R*<sub>2</sub> measured solar radiation on horizontal surface (if available), cal/cm<sup>2</sup>/day.
- R*<sub>3</sub> potential solar radiation on a horizontal surface, cal/cm<sup>2</sup>/day.
- R*<sub>4</sub> potential solar radiation on a sloping surface, cal/cm<sup>2</sup>/day.
- R*<sub>5</sub> estimated solar radiation on a sloping surface, cal/cm<sup>2</sup>/day.
- R*<sub>6</sub> estimated solar radiation on a sloping surface based on map area, cal/cm<sup>2</sup>/day.

- $T$  temporary variable, hour angle of sunset.  
 $T_0$  hour angle of sunrise on horizontal surface.  
 $T_1$  hour angle of sunset on horizontal surface.  
 $T_2$  hour angle of sunrise on slope.  
 $T_3$  hour angle of sunset on slope.  
 $T_4$  number of hours between sunrise on slope and solar noon.  
 $T_5$  number of hours between solar noon and sunset on slope.  
 $T_6$  hour angle of sunrise on equivalent slope.  
 $T_7$  hour angle of sunset on equivalent slope.  
 $T_8$  hour angle of second sunrise on slope.  
 $T_9$  hour angle of second sunset on slope.  
 $V, W, X, Y, Z$  temporary variables for function subroutines.

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